

NAG Toolbox for MATLAB

g13ac

1 Purpose

g13ac calculates partial autocorrelation coefficients given a set of autocorrelation coefficients. It also calculates the predictor error variance ratios for increasing order of finite lag autoregressive predictor, and the autoregressive parameters associated with the predictor of maximum order.

2 Syntax

```
[p, v, ar, nvl, ifail] = g13ac(r, nl, 'nk', nk)
```

3 Description

The data consist of values of autocorrelation coefficients $r_1, r_2, \dots, r_K$, relating to lags $1, 2, \dots, K$. These will generally (but not necessarily) be sample values such as may be obtained from a time series x_t using g13ab.

The partial autocorrelation coefficient at lag l may be identified with the parameter $p_{l,l}$ in the autoregression

$$x_t = c_l + p_{l,1}x_{t-1} + p_{l,2}x_{t-2} + \dots + p_{l,l}x_{t-l} + e_{l,t}$$

where $e_{l,t}$ is the predictor error.

The first subscript l of $p_{l,l}$ and $e_{l,t}$ emphasises the fact that the parameters will in general alter as further terms are introduced into the equation (i.e., as l is increased).

The parameters are determined from the autocorrelation coefficients by the Yule–Walker equations

$$r_i = p_{i,1}r_{i-1} + p_{i,2}r_{i-2} + \dots + p_{i,i}r_{i-i}, \quad i = 1, 2, \dots, l$$

taking $r_j = r_{|j|}$ when $j < 0$, and $r_0 = 1$.

The predictor error variance ratio $v_l = \text{Var}(e_{l,t}) / \text{Var}(x_t)$ is defined by

$$v_l = 1 - p_{l,1}r_1 - p_{l,2}r_2 - \dots - p_{l,l}r_l.$$

The above sets of equations are solved by a recursive method (the Durbin–Levinson algorithm). The recursive cycle applied for $l = 1, 2, \dots, (L-1)$, where L is the number of partial autocorrelation coefficients required, is initialized by setting $p_{1,1} = r_1$ and $v_1 = 1 - r_1^2$.

Then

$$\begin{aligned} p_{l+1,l+1} &= (r_{l+1} - p_{l,1}r_l - p_{l,2}r_{l-1} - \dots - p_{l,l}r_1) / v_l \\ p_{l+1,j} &= p_{l,j} - p_{l+1,l+1}p_{l,l+1-j}, \quad j = 1, 2, \dots, l \\ v_{l+1} &= v_l(1 - p_{l+1,l+1})(1 + p_{l+1,l+1}). \end{aligned}$$

If the condition $|p_{l,l}| \geq 1$ occurs, say when $l = l_0$, it indicates that the supplied autocorrelation coefficients do not form a positive-definite sequence (see Hannan 1960), and the recursion is not continued. The autoregressive parameters are overwritten at each recursive step, so that upon completion the only available values are $p_{l,j}$, for $j = 1, 2, \dots, L$, or $p_{l_0-1,j}$ if the recursion has been prematurely halted.

4 References

Box G E P and Jenkins G M 1976 *Time Series Analysis: Forecasting and Control* (Revised Edition) Holden-Day

Durbin J 1960 The fitting of time series models *Rev. Inst. Internat. Stat.* **28** 233

Hannan E J 1960 *Time Series Analysis* Methuen

5 Parameters

5.1 Compulsory Input Parameters

- 1: **r(nk)** – double array
The autocorrelation coefficient relating to lag k , for $k = 1, 2, \dots, K$.
- 2: **nl** – int32 scalar
 L , the number of partial autocorrelation coefficients required.
Constraint: $0 < \mathbf{nl} \leq \mathbf{nk}$.

5.2 Optional Input Parameters

- 1: **nk** – int32 scalar
Default: The dimension of the array **r**.
 K , the number of lags. The lags range from 1 to K and do not include zero.
Constraint: $\mathbf{nk} > 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

- 1: **p(nl)** – double array
 $\mathbf{p}(l)$ contains the partial autocorrelation coefficient at lag l , $p_{l,l}$, for $l = 1, 2, \dots, \mathbf{nl}$.
- 2: **v(nl)** – double array
 $\mathbf{v}(l)$ contains the predictor error variance ratio v_l , for $l = 1, 2, \dots, \mathbf{nl}$.
- 3: **ar(nl)** – double array
The autoregressive parameters of maximum order, i.e., p_{Lj} if **ifail** = 0, or $p_{l_0-1,j}$ if **ifail** = 3, for $j = 1, 2, \dots, \mathbf{nl}$.
- 4: **nl** – int32 scalar
The number of valid values in each of **p**, **v** and **ar**. Thus in the case of premature termination at iteration l_0 (see Section 3), **nl** is returned as $l_0 - 1$.
- 5: **ifail** – int32 scalar
0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail = 1

On entry, $\mathbf{nk} \leq 0$,
or $\mathbf{nl} \leq 0$,
or $\mathbf{nk} < \mathbf{nl}$.

ifail = 2

On entry, the autocorrelation coefficient of lag 1 has an absolute value greater than or equal to 1.0; no recursions could be performed.

ifail = 3

Recursion has been prematurely terminated; the supplied autocorrelation coefficients do not form a positive-definite sequence (see Section 3). Parameter **nv1** returns the number of valid values computed.

7 Accuracy

The computations are believed to be stable.

8 Further Comments

The time taken by g13ac is proportional to $(\mathbf{nv1})^2$.

9 Example

```
r = [0.8004;  
     0.4355;  
     0.0328;  
    -0.2835;  
    -0.4505;  
    -0.4242;  
    -0.2419;  
    -0.055;  
     0.3783;  
     0.5857];  
nl = int32(5);  
[p, v, ar, nv1, ifail] = g13ac(r, nl)  
  
p =  
    0.8004  
   -0.5708  
   -0.2388  
   -0.0494  
   -0.0321  
v =  
    0.3594  
    0.2423  
    0.2284  
    0.2279  
    0.2276  
ar =  
    1.1076  
   -0.2899  
   -0.1925  
   -0.0138  
   -0.0321  
nv1 =  
      5  
ifail =  
      0
```